Consider the conic with the polar equation $r = \dfrac{32}{9 - 7\cos\theta}$. **SCORE: _____ / 30 PTS**

[a] What is the shape of the graph of the equation ?

$$r = \frac{\frac{32}{9}}{1 - \frac{7}{9}\cos\theta} \qquad e = \frac{7}{9} < 1 \longrightarrow \text{ELLIPSE}$$

[b] Find the equation of the directrix.

$$ep = \frac{32}{9}$$

$$\frac{7}{9}p = \frac{32}{9} \qquad\qquad x = -\frac{32}{7}$$

$$p = \frac{32}{7}$$

[c] Find the rectangular co-ordinates of the focus/foci, using the process in the lecture & website handout.

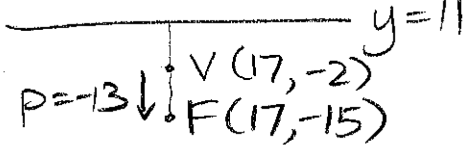
θ	r	(x,y)
0	16	$(16,0)$
$\frac{\pi}{2}$	$\frac{32}{9}$	$(0,\frac{32}{9})$
π	2	$(-2,0)$
$\frac{3\pi}{2}$	$\frac{32}{9}$	$(0,-\frac{32}{9})$

VERTICES

$$\text{CENTER} = \left(\frac{16-2}{2}, 0\right) = (7,0)$$

$$\text{FOCI} = (14,0),\ (0,0)$$

Find the equation of the parabola with focus $(17, -15)$ and directrix $y = 11$. **SCORE: _____ / 15 PTS**



$$\text{VERTEX} = \left(17, \frac{-15+11}{2}\right) = (17, -2)$$

$$p = -15 - -2 = -13$$

$$(x-17)^2 = 4(-13)(y+2)$$

$$(x-17)^2 = -52(y+2)$$

Consider the polar equation $r = -5 - 4\sin\theta$. **SCORE: _____ / 30 PTS**

The following symmetry tests all fail: $(-r, \theta)$, $(-r, -\theta)$ **and** $(-r, \pi - \theta)$

[a] Run the other standard tests for symmetry for the polar equation, and summarize all conclusions in the table below.

SYMMETRIC OVER $\theta = \frac{\pi}{2}$?	YES
SYMMETRIC OVER POLE ?	NO CONCLUSION
SYMMETRIC OVER POLAR AXIS ?	NO CONCLUSION

$\theta = \frac{\pi}{2}$: $(r, \pi - \theta)$

$r = -5 - 4\sin(\pi - \theta)$

$r = -5 - 4[\sin\pi\cos\theta - \cos\pi\sin\theta]$

$r = -5 - 4\sin\theta$

POLE: $(r, \pi + \theta)$

$r = -5 - 4\sin(\pi + \theta)$

$r = -5 - 4[\sin\pi\cos\theta + \cos\pi\sin\theta]$

$r = -5 + 4\sin\theta$

POLAR AXIS: $(r, -\theta)$

$r = -5 - 4\sin(-\theta)$

$r = -5 + 4\sin\theta$

[b] What is the minimum interval of θ – values that must be plotted before using symmetry to complete the graph ?

$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

[c] Name the shape of the graph of the polar equation. If the graph is a rose curve, state the number of petals.

$|-4| < |-5| < 2|-4| \longrightarrow$ LIMAÇON WITH DIMPLE

Convert the rectangular equation $x = 3y - 5$ to polar. **SCORE: _____ / 15 PTS**

$r\cos\theta = 3r\sin\theta - 5$

$r\cos\theta - 3r\sin\theta = -5$

$r(\cos\theta - 3\sin\theta) = -5$

$$r = -\frac{5}{\cos\theta - 3\sin\theta}$$

$$r = \frac{5}{3\sin\theta - \cos\theta}$$

Consider the ellipse with foci $(9, -10)$ and $(9, 2)$ and minor axis of length 8. **SCORE: _____ / 20 PTS**

[a] Find the equation of the ellipse.

CENTER $= \left(9, \frac{-10+2}{2}\right) = (9, -4)$

$2b = 8 \longrightarrow b = 4$

$c = 2 - {}^{-}4 = 6$

$a^2 = 4^2 + 6^2 = 52$

$$\frac{(x-9)^2}{4^2} + \frac{(y+4)^2}{52} = 1 \longrightarrow \frac{(x-9)^2}{16} + \frac{(y+4)^2}{52} = 1$$

- V
- F(9, 2)
- C(9, −4)
- F(9, −10)
- V

[b] Find the co-ordinates of the vertices.

$(9, -4 \pm 2\sqrt{13})$

Pat & Reese live in separate houses on Taylor Street.
Taylor Street is parallel to Jordan Road. Both roadways are straight.

SCORE: _____ / 10 PTS

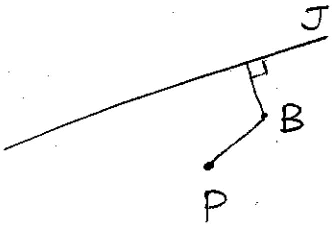
<u>Draw diagrams and write algebraic equations involving distances to answer the following questions.</u>

[a] A bird is flying through the neighborhood. The bird is always 300 meters closer to Pat's house than to ~~Chris's~~ REESE'S house. What is the shape of the bird's path ? (Assume the bird is flying at a constant low height.)

$BR - BP = 300$

HYPERBOLA

[b] There is a bike path nearby. Every point on the path is three times as far from Jordan Road as it is from Pat's house. What is the shape of the bike path ?



$BJ = 3 \times BP$

$e = \frac{1}{3} = \frac{BP}{BJ}$

ELLIPSE

Consider the conic with equation $2x^2 - 3y^2 + 8x + 18y - 1 = 0$. **SCORE: _____ / 30 PTS**

[a] Find the co-ordinates of the focus/foci.

$$2x^2 + 8x - 3y^2 + 18y = 1$$

$$2(x^2+4x+4) - 3(y^2-6y+9) = 1 + 8 - 27$$

$$2(x+2)^2 - 3(y-3)^2 = -18$$

$$\frac{(y-3)^2}{6} - \frac{(x+2)^2}{9} = 1 \qquad c^2 = 6 + 9 = 15$$

$$(-2,\ 3 \pm \sqrt{15})$$

[b] If the equation corresponds to a circle, find its radius.
If the equation corresponds to a parabola, find its vertex & directrix.
If the equation corresponds to an ellipse, find the endpoints of its minor axes.
If the equation corresponds to a hyperbola, find the equations of the asymptotes.

$$y - 3 = \pm \frac{\sqrt{6}}{3}(x+2)$$